

Advanced Collisions

Teacher's Guide

1.0 Summary

Advanced Collisions is the seventh activity in the Dynamica sequence. This activity should be done after *Collisions and Momentum in 1D* and should take students approximately 30 minutes. This is a more challenging activity that includes collisions in two-dimensions. Students may want to use a calculator during this activity.

2.0 Learning Goals

Driving Question: When objects collide in different situations, what happens?

This activity presents models of two balls colliding, three balls colliding, sticky collisions and offset collisions.

Step One: **Calculating Momentum** reviews the concept of total momentum of a system. Students practice calculating p_{total} .

Step Two: **Three Ball Game** presents the students with a mystery event that can be understood by using what they learned about conservation of momentum.

Step Three: **Inelastic Collisions** displays an inelastic collision, where the two balls stick together. Students are encouraged to test their existing momentum rules.

Step Four: **Collisions in Two Dimensions** allows students to observe that the same rules can be applied to the X and Y directions independently.

Additional Teacher Background

Momentum is of central importance in physics and working with it may be the first time that students work with a **derived** quantity, which is not measured directly. Rather, momentum is the product of two measurable quantities - mass and velocity. The activity helps them pay attention to the value of the product (mv) rather than the values of the components.

The reason momentum is so important is that momentum is **conserved**, meaning that the amount of momentum in a given direction is the same before and after an event. This rule helps with the solution to a great number of problems.

Momentum often seems to disappear. For instance, when a ball bounces on the ground it appears to change from mv to $-mv$ because it reverses direction. Where did the $2mv$ go? In fact, the earth moved a little bit, but imperceptibly because it's so massive! When a car slows down, its decrease in momentum is exactly matched by an increase in the earth's momentum in the opposite direction. We trust the conservation of momentum to be true even when we can't measure it.

Energy and momentum are related, but they are not the same thing. Momentum (mv) is a vector in the same direction as the velocity. Kinetic energy ($1/2 m v^2$) depends on the square of the velocity. It is a scalar quantity. It is not a vector and

does not have direction. It also has other forms, such as potential energy and heat (see the *Energy* activity).

Students may be surprised that momentum is conserved even in inelastic collisions, such as when two balls stick together (step six). This may be confusing because the kinetic energy of the balls ($\frac{1}{2} m v^2$) is **not** conserved. Some of the energy is converted to heat when the balls collide. If this energy is included, total energy **is** conserved.

Another surprising thing is that both balls can have momentum but the **total** momentum of the system is still zero. Momentum is a vector, like force and velocity; the momentum of different objects can cancel each other.

Additional Activities

Collisions and momentum: Use two sonar ranglers to explore the before-and-after velocities of low-friction carts. Try changing their relative masses and calculating the total momentum before and after the collision. Is momentum conserved? Try inelastic collisions, where the two carts stick together. Is momentum conserved then?

3.0 Standards Alignment

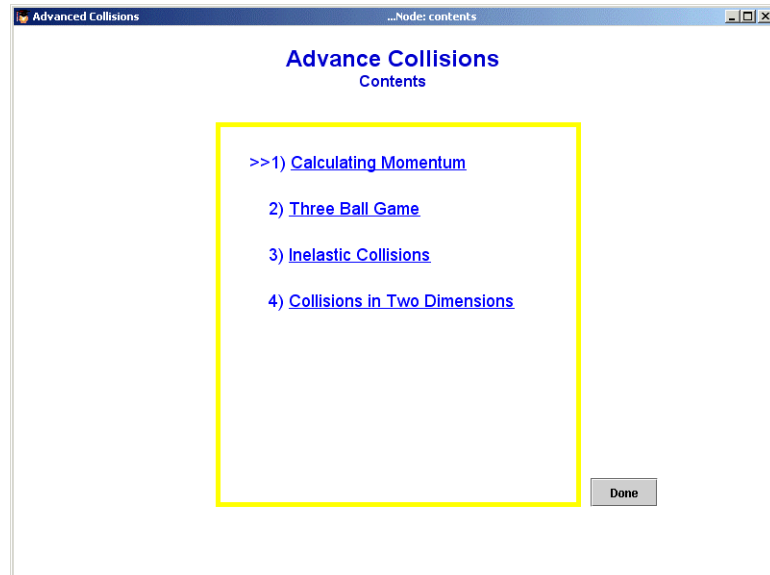
Alignment to National Math and Science Standards (NCTM or NSES)

Objective	Standards
Students will learn the definition of momentum as a vector quantity.	<ul style="list-style-type: none">• Students should develop an understanding of properties of, and representations for, the addition and multiplication of vectors.
Students will calculate the momentum of moving objects.	<ul style="list-style-type: none">• Students should interpret representations of functions of two variables.• Students should develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
Students will predict the outcome of an event and compare it with data from a simulation.	<ul style="list-style-type: none">• Students should draw reasonable conclusions about a situation being modeled.• Students should recognize reasoning and proof as fundamental aspects of mathematics.
Students will use conservation of momentum to predict what happens when objects collide.	<ul style="list-style-type: none">• Students should identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.• Students should use the language of mathematics to express mathematical ideas precisely.
Students will compare elastic and inelastic collisions.	<ul style="list-style-type: none">• Students should recognize and apply mathematics in contexts outside of mathematics.
Students will analyze X and Y components independently in two-dimensional collisions.	<ul style="list-style-type: none">• Students should analyze precision, accuracy, and approximate error in measurement situations.• Students should select, apply, and translate among mathematical representations to solve problems.

4.0 Activity Sections

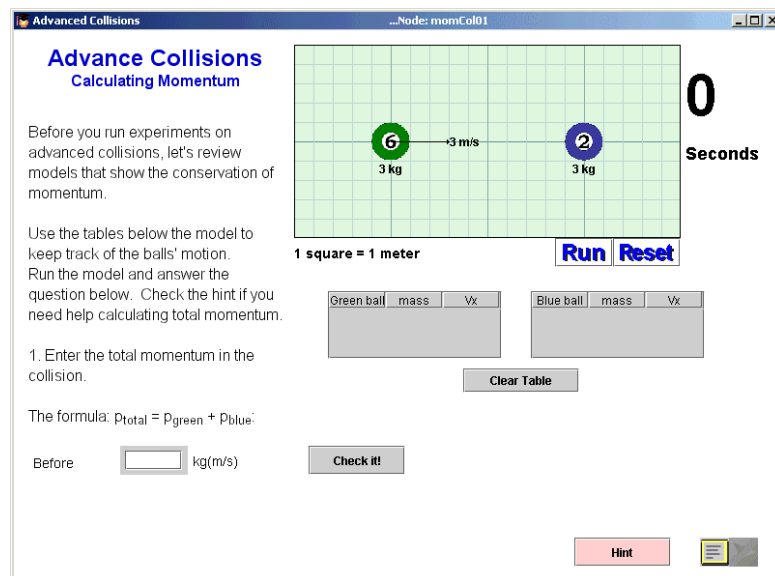
4.1 Table of Contents

This activity has 4 sections.



4.2 1) Calculating Momentum

In this section, students review how to calculate the total momentum of a system before and after a collision. Of course, the models depict a frictionless environment.



Calculate total momentum

Advanced Collisions ...Node: momCol03

Advance Collisions

Calculating Momentum

Set the velocity of the green and blue balls such that the total momentum is 0 kg(m/s). Run the model.

3. Is momentum conserved in this example?
 Yes
 No

4. How can you tell that momentum is/is not conserved?

1 square = 1 meter

0 Seconds

39 m/s

5 kg 3 kg

Run Reset

Green Ball Blue Ball

-10 -8 -6 -4 -2 0 2 4 6 8 10 -10 -8 -6 -4 -2 0 2 4 6 8 10

Green ball	mass	Vx	Blue ball	mass	Vx

Clear Table

Hint

Setting the green ball to 3m/s and blue to -5m/s results in 0 momentum

4.3 2) Three Ball Game

In this section, students run models of collisions with a “mystery” ball. This is a third, invisible ball.

Advanced Collisions ...Node: game1

Advance Collisions

Mystery Game

Did the balls do something unexpected? Can you think of a way to explain their behavior? Run it again and watch carefully.

5. What do you think is going on here?

1 square = 1 meter

3 m/s

1 kg 1 kg

Pause Reset

	Green	Blue
Mass	1	1
Initial Velocities	3	0
Final Velocities	0	3

Run the model and answer the question.

Advanced Collisions ...Node: game1 a

Advance Collisions Mystery Game

You may have guessed that there was a hidden object. Now you can see it - a red ball with the same mass as the others. Run it again.

1 square = 1 meter **Run Reset**

	Green	Blue
Mass	1	1
Initial Velocities	3	0
Final Velocities		

The mystery ball is revealed.

Advanced Collisions ...Node: hiddenBallGame3

Advance Collisions Mystery Game

You may have guessed that there's a hidden ball. What can you figure out about it? Look at the hints if you need to.

8. What is the mass of the hidden ball?

9. Is the hidden ball moving after the collisions? How can you tell?

1 square = 1 meter **Run Reset**

	Green	Blue
Mass	2	3
Initial Velocities	3	0
Final Velocities	0	0

Hint

Another hidden ball example

4.4 3) Inelastic Collisions

This section introduces students to totally inelastic collisions, or sticky collisions. Students are presented with a few different examples and are asked to calculate the initial and final momentum in a collision as well as the final velocity of the collision.

Advanced Collisions ...Node: stickyCollision1

Advance Collisions

Inelastic Collisions

This model shows a new type of collision, a sticky collision. Can you guess where this name comes from? That's right - the two balls stick together when they collide, like clay or chewing gum. Run the model.

17. Do you think momentum is conserved, even if they stick together? Why?

1 square = 1 meter

Pause Reset

	Green	Blue
Mass	1	1
Initial Velocities	3	0
Final Velocities	1.5	1.5

Conservation in a sticky collision

Advanced Collisions ...Node: stickyCollision2-1

Advance Collisions

Inelastic Collisions

That's right! You get a piece of pie. Go on.

1 square = 1 meter

Pause Reset

21. Initial momentum: kg m / s ✓

22. Final momentum: kg m / s ✓

23. Final velocity: m/s ✓

	Green	Blue
Mass	1	1
Initial Velocities	5	1

Calculate the final velocity.

Advanced Collisions ...Node: stickyCollision3-1

Advance Collisions

Inelastic Collisions

Now the balls are NOT the same mass. Can you still use the conservation of momentum rule to predict the final velocity of the balls stuck together? Give it a try.

1 square = 1 meter **Run** **Reset**

28. Initial momentum: kg m / s **Check it!**

29. Final momentum: kg m / s **Check it!**

30. Final velocity: m/s **Check it!**

	Green	Blue
Mass	2	1
Initial Velocities	2	-4

Balls of different mass

Advanced Collisions ...Node: stickyCollision3-2

Advance Collisions

Inelastic Collisions

That's right! You get a piece of pie. Go on.

1 square = 1 meter **Pause** **Reset**

31. Initial momentum: kg m / s **Check it!** ✓

32. Final momentum: kg m / s **Check it!** ✓

33. Final velocity: m/s **Check it!** ✓

	Green	Blue
Mass	2	1
Initial Velocities	2	-1

Final velocity is 1 m/s

4.5 4) Collisions in Two Dimensions

Offset collisions adhere to the Law of Conservation of Momentum. In this section, students will run models of 2D collisions where they can adjust the offset as well as the mass and velocity of the balls. Students are asked to compute momentum in both the x and y directions. A calculator may be used in this section. Students can also use the built in Windows calculator found in the Start menu, under Programs and Accessories.

Advanced Collisions

Advance Collisions In two dimensions

In this section, we will explore collisions in 2 dimensions.

Notice that the balls are offset, which means that they will not collide head-on. Run the model and observe the motion of the balls.

Look at the table. Notice that before the collision, both balls have a velocity in the x direction. After the collision, they have a velocity in both the x and y directions.

Go on to learn how momentum is conserved in an offset collision.

Green ball	mass	Vx
trial 2		
Before	4.0	3.0
After	4.0	-1.5

Blue ball	mass	Vx
trial 2		
Before	4.0	-3.0
After	4.0	1.5

Data tables show velocity before and after the collisions.

Advanced Collisions ...Node: 2D_02

Advance Collisions In two dimensions

Momentum is still conserved, even if the collision is offset. In the example below, the balls both have an initial velocity in the x dimension and therefore, a momentum in the x direction. They do not have an initial momentum in the y direction.

To calculate the initial momentum in the x direction, use the $p = mv$ formula that you learned in the previous activity. In the example below, the initial momentum in the y direction is $0 \text{ kg}\cdot\text{m/s}$ because there is no initial velocity in the y direction.

Enter the values for the momentums BEFORE the collision:

36. Momentum in the x direction kg(m/s) Check it!

37. Momentum in the y direction kg(m/s) Check it!

38. Total momentum before collision kg(m/s) Check it!

Go on to learn how momentum is conserved in an offset collision.

Introduction to momentum in the y-direction

Advanced Collisions ...Node: twoDim6_2

Advance Collisions

In two dimensions

Look at the table. Now there is a V_x and a V_y . You may use a calculator to answer these questions.

41. Final Momentum in the X-direction

42. Final Momentum in the Y-direction

1 square = 1 meter

Green Ball Offset Blue Ball

Initial Values Vx(m/s) 3 Mass(g) 4 Vx(m/s) -2 Mass(kg) 4

Data Table

trial 1	mass	Vx	Vy
Before	4	3	0
After	4	-0.2	2.4
trial 2			

trial 1	mass	Vx	Vy
Before	4	-2	0
After	4	1.2	-2.4
trial 2			

current trial 1

Use tables to calculate momentum in x and y directions.

Advanced Collisions ...Node: twoDim6_4

Advance Collisions

In two dimensions

Look at the table. Now there is a V_x and a V_y . You may use a calculator to answer these questions.

45. Final Momentum in the X-direction

46. Final Momentum in the Y-direction

1 square = 1 meter

Green Ball Offset Blue Ball

Initial Values Vx(m/s) 2 Mass(g) 1 Vx(m/s) -2 Mass(kg) 2

Data Table

trial 1	mass	Vx	Vy
Before	1	2	0
After	1	-1.41	2.56
trial 2			

trial 1	mass	Vx	Vy
Before	2	-2	0
After	2	-0.3	-1.28
trial 2			

current trial 2

Adjust the mass of the balls and calculate the final momentum.

Advanced Collisions ...Node: twoDim8

Advance Collisions

In two dimensions

Good job!
Click the Next arrow to go on.

Initial Momentum in the X-direction: 0
Initial Momentum in the Y-direction: 0
Final Momentum in the X-direction: 0
Final Momentum in the Y-direction: 0

1 square = 1 meter

Green Ball: V_x (m/s) 6, Mass(g) 3
Blue Ball: V_x (m/s) -3, Mass(kg) 6

Offset: -10 -6 -2 2 6 10

Pause Reset

trial 2	mass	Vx	Vy
Before	3	6	0
After	3	-4.08	-4.4
trial 3			

trial 2	mass	Vx	Vy
Before	6	-3	0
After	6	2.04	2.2
trial 3			

Clear Table

Set the model to have the same

Advanced Collisions ...Node: twoDimNote

Advance Collisions

In two dimensions

Now the target ball is invisible, and it starts at rest, and you don't know its mass. Can you figure out its mass?

47. How will you figure it out?(Hint: use conservation of momentum.)

1 square = 1 meter

Green Ball: V_x (m/s) 2, Mass(g) 4
Blue Ball: V_x (m/s) -2, Mass(kg) ?

Offset: -10 -6 -2 2 6 10

trial 1	mass	Vx	Vy
current trial 1			

trial 1	mass	Vx	Vy
current trial 1			

Clear Table

Now, the second ball is invisible with a mystery mass

The next problem is the biggest challenge. Students must calculate the mass of a ball based on the fact that the total momentum of the collision is the same before and after. In this example, the total initial momentum is 8. Use $p = mv$ to find that the mystery ball is 2kg.

Advanced Collisions ...Node: twoDim9

Advance Collisions

In two dimensions

Look at the table. Now there is a V_x and a V_y . You may use a calculator to answer these questions.

48. The mass of the hidden ball.

1 square = 1 meter

Green Ball **Blue Ball**

Initial Values

Green Ball: V_x (m/s) = 2, Mass(g) = 4

Blue Ball: V_x (m/s) = 0, Mass(kg) = ?

Offset: -10 -6 -2 2 6 10

Data Table

trial	mass	V_x	V_y
trial 1			
Before	4	2	0
After	4	1.15	0.64
trial 2			

trial	mass	V_x	V_y
trial 1			
Before	?	0	0
After	?	1.71	-1.28
trial 2			

Use the $p=mv$ formula to calculate the mass.

Advanced Collisions ...Node: twoDimEnd

Advance Collisions

In two dimensions

Success!

You have completed the Advanced Collisions activity. The next activity is "Balancing Forces."

Final screen of activity

5.0 Student Reports

Your students' work with the *Advanced Collisions* activity is logged and viewable on the MAC Project Web Portal at <http://mac.concord.org>. For each student, you can view a report containing questions and student answers.

The next activity in the sequence should be *Balancing Forces*.